$$S = \frac{3}{2}\int_0^1 \frac{t^{1/2}(1-t)}{2-(1-t)r}\,dt.$$

When $r = 0$, we find $S = 1/5$, and when $r = 1$, the integral reduces to $5 - (3\pi/2)$; we do not know of any evaluation of the series independent of the integral calculus.

Also solved by H. F. GUMMER and E. H. CLARKE.

2709 [June, 1918]. Proposed by E. V. HUNTINGTON, Harvard University.

The following problem was suggested to the proposer by a professor of biology, who has found the result useful in certain problems concerning the equilibrium of chemical reactions. Starting with

$$\mu(c_1 + y - x)(c_2 - x)(c_3 - x)\cdots(c_n - x) = \lambda(b_1 + x)(b_2 + x)(b_3 + x)\cdots(b_m + x),$$

where $\mu c_1 c_2 c_3 \cdots c_n = \lambda b_1 b_2 b_3 \cdots b_m$ (all the letters being positive), find the limit of x/y as y approaches zero; and show that for small values of y, the value of x/y is always *less* than this limit.

I. SOLUTION BY THE PROPOSER.

Let $y = f(x) = f(0) + f'(0)\cdot x + \frac{1}{2}f''(0)\cdot x^2 + \cdots$.

First. From the given equations, when $x = 0$, $y = 0$; hence $f(0) = 0$.

Second. Taking logarithms and differentiating, we have

$$\frac{f'(x)-1}{c_1+y-x} = \left(\frac{1}{c_2-x}+\frac{1}{c_3-x}+\cdots+\frac{1}{c_n-x}\right)+\left(\frac{1}{b_1+x}+\frac{1}{b_2+x}+\frac{1}{b_3+x}+\cdots+\frac{1}{b_m+x}\right),$$

whence

$$\frac{f'(0)}{c_1} = \left(\frac{1}{c_1}+\frac{1}{c_2}+\frac{1}{c_3}+\cdots+\frac{1}{c_n}\right)+\left(\frac{1}{b_1}+\frac{1}{b_2}+\frac{1}{b_3}+\cdots+\frac{1}{b_m}\right).$$

Third. Differentiating again, we have

$$\frac{(c_1+y-x)f''(x)-[f'(x)-1]^2}{(c_1+y-x)^2} = \left[\frac{1}{(c_2-x)^2}+\frac{1}{(c_3-x)^2}+\cdots+\frac{1}{(c_n-x)^2}\right] - \left[\frac{1}{(b_1+x)^2}+\frac{1}{(b_2+x)^2}+\frac{1}{(b_3+x)^2}+\cdots+\frac{1}{(b_m+x)^2}\right],$$

whence

$$\frac{f''(0)}{c_1} = \left[\left(\frac{1}{c_2}+\frac{1}{c_3}+\cdots+\frac{1}{c_n}\right)+\left(\frac{1}{b_1}+\frac{1}{b_2}+\frac{1}{b_3}+\cdots+\frac{1}{b_m}\right)\right]^2 + \left[\frac{1}{c_2^2}+\frac{1}{c_3^2}+\cdots+\frac{1}{c_n^2}\right]-\left[\frac{1}{b_1^2}+\frac{1}{b_2^2}+\frac{1}{b_3^2}+\cdots+\frac{1}{b_m^2}\right],$$

which is clearly *positive*.

Fourth. With these values of $f(0)$, $f'(0)$, and $f''(0)$, we have

$$\frac{y}{x} = f'(0) + \frac{f''(0)}{2}\cdot x + \cdots, \qquad \text{or} \qquad \frac{x}{y} = \frac{1}{f'(0)} - \frac{f''(0)}{2[f'(0)]^2}\cdot x + \cdots,$$

from which the required solution is obvious.

Note: It is clear that x and y have like signs for small values of x since $f'(0) > 0$.

II. SOLUTION BY A. M. HARDING, University of Arkansas.

Solving the given equation for y gives

$$y = x - c_1 + \frac{\lambda(b_1+x)(b_2+x)\cdots(b_m+x)}{\mu(c_2-x)(c_3-x)\cdots(c_n-x)} = x - c_1 + \frac{c_1(1+x/b_1)(1+x/b_2)\cdots(1+x/b_m)}{(1-x/c_2)(1-x/c_3)\cdots(1-x/c_n)}.$$

Let c_k denote the smallest of $c_2, c_3, \cdots c_n$. Then, if $x < c_k$, each of the expressions $\frac{1}{1-x/c_i}$, $(i = 2, 3, \cdots, n)$, may be expanded into a convergent series and we obtain, after reduction,

$$y = Ax + Bx^2 + Cx^3 + Dx^4 + \cdots, \tag{1}$$

where

$$A = 1 + c_1 \sum_{i=1}^{m} \frac{1}{b_i} + c_1 \sum_{i=2}^{n} \frac{1}{c_i} = c_1 \left[\sum_{i=1}^{m} \frac{1}{b_i} + \sum_{i=1}^{n} \frac{1}{c_i} \right]$$

and $B, C, D, \cdots$ are all positive.

Since each of the series formed from $\dfrac{1}{1 - x/c_i}$ converges in the interval $0 \leqq x < c_k$, the power series (1) will converge in this interval. And, since in (1) A is not zero, there is one and only one solution for x in terms of y, and this solution may be obtained as a power series in y convergent for sufficiently small values of y.

Let

$$x = \alpha y + \beta y^2 + \gamma y^3 + \delta y^4 + \cdots. \tag{2}$$

Substitute (2) in (1), equate coefficients of like powers of y, and obtain

$$\alpha = \frac{1}{A}, \qquad \beta = -\frac{B}{A^3}, \qquad \gamma = \frac{2B^2}{A^5} - \frac{C}{A^4}, \cdots.$$

That is

$$x = \frac{1}{A} y - \frac{B}{A^3} y^2 + \left(\frac{2B^2}{A^5} - \frac{C}{A^4} \right) y^3 + \cdots$$

or

$$\frac{1}{A} - \frac{x}{y} = \frac{B}{A^3} y - \left(\frac{2B^2}{A^5} - \frac{C}{A^4} \right) y^2 + \cdots. \tag{3}$$

Thus

$$\lim_{y \to 0} \frac{x}{y} = \frac{1}{A} = \frac{1}{c_1 \left(\sum_{i=1}^{m} \frac{1}{b_i} + \sum_{i=1}^{n} \frac{1}{c_i} \right)}.$$

For sufficiently small values of y the right member of (3) is positive. Hence, for small values of y, the value of x/y is always less than $1/A$.

NOTES AND NEWS.

Edited by E. J. Moulton, Northwestern University, Evanston, Ill.

Dr. G. H. Light has been promoted from assistant professor to associate professor of mathematics at the University of Colorado.

The fellows in mathematics at the University of Chicago for next year are Miss Gladys Gibbens, Mrs. Mayme I. Logsdon, and Mr. Frank E. Wood.

Dr. C. C. Camp, who recently returned from war service in France, has been appointed instructor in mathematics at the University of Illinois.

At Brown University Captain R. W. Burgess, now of the Bureau of Statistics, Washington, and Dr. R. E. Gilman have been appointed assistant professors of mathematics, and Mr. C. R. Adams instructor.

At Cornell University Assistant Professor F. R. Sharpe has been promoted to a full professorship and Mr. V. G. Grove, of the University of Kentucky, has been appointed instructor.